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MESA

Mathematics, Science and Technology Institute

June 30th through July 11th, 2008

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Agenda & Curriculum



www.InnovateCalifornia.net



Funding provided by Department of Labor, Employment & Training Administration: WIRED Initiative

Week 1
Monday Agenda

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| 9 AM | Welcome and Business: sign in, introductions, notifications re: attendance requirements and equipment ownership, W-9, and overview of institute |
| 10AM | <p>Celsius vs. Fahrenheit Activity</p> <p><i>Goal:</i> Participants will learn to use the EASYLINK with the temperature probe, input data, graph data, and fit lines to data using TI-84. Participants will review parameters a and b in $y = ax + b$ and the conversion formula for Fahrenheit to Celsius.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants measure the temperature of several objects around the room, using the temperature probe with the EASYLINK. • Participants graph data on graph paper, and fit line of best fit with eye and ruler. They note the slope and the y-intercept. • Participants are led to enter data in lists, plot data using STAT PLOT, and finally approximate a line of best fit, using Transformation Graphing. • Group discusses meaning of the parameters, a and b in the equation, $y = ax + b$. |
| 12 PM | Lunch |
| 1 PM | <p>Hooke's Law Activity</p> <p><i>Goal:</i> Participants will review $y = ax + b$ and Hooke's Law for linear springs. Participants will also review entering and graphing data on the TI 84.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will record the displacement of a rubberband as the weight in a bucket it holds is increased. • Participants will enter Force vs. displacement data into their TI 84, and approximate a line of best fit. • Participants will review the meaning of a and b in the equation $y = ax + b$ |

Week 1
Tuesday Agenda

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| 9 AM | <p>Intro 1D Motion</p> <p><i>Goal:</i> Participants will learn definitions of displacement, distance traveled, speed, velocity, and acceleration.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Lim provides direct instruction with participants taking notes |
| 10 AM | <p>Walk This Way 1</p> <p><i>Goal:</i> Introduce participants to concepts of velocity and displacement: 1D Motion. Introduce participants to CBRanger and grouping data on the calculator. Review $y = ax+b$. Review inputting and graphing data on the TI 84.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants mark off 5m, a point of reference, and an initial position point. 1st: One group member starts at the initial position point and walks away at a constant speed, while another member measures their displacement with a CBR. Participants save this data by grouping. 2nd: Repeat with the walker traveling at an increased but still constant speed. Participants save this data by grouping as well. 3rd: Repeat with the walker starting at the 5m mark and traveling toward the point of reference at a constant speed. Participants save this data by grouping as well. • Participants enter data into their lists and graph all three sets of data on the same axes. • Participants approximate a line of best fit to each graph, using Transformation Graphing. • Calahan reviews the meaning of m and b in this application. |
| 12 PM | Lunch |
| 1 PM | <p>Walk this Way 2</p> <p><i>Goal:</i> Introduce participants to a graphical representation of velocity, displacement, distance traveled, and acceleration via the CBRanger.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Calahan asks participant to walk away from CBRanger and return and then discusses velocity, displacement, distance traveled, and acceleration on the trip. • Calahan asks participant to walk away from CBR at a constant speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. • Calahan asks participant to walk away from CBR at an increasing speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. • Calahan asks participant to walk away from CBR at a decreasing speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. |

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| 2:00 PM | <p>Walk-A-Graph</p> <p><i>Goal:</i> Participants practice concepts via graphical interpretation of velocity and displacement. Participants practice using the CBRanger.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none">• Participants are given a position vs. time graph and discuss the type of motion that would create that graph. They record their discussion and then one participant attempts to walk out the motion, while the other pair member checks their result with the CBRanger.• Repeat with a new given displacement, but pair members switch roles. Continue till all possibilities “walked.” |
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Week 1
Thursday Agenda

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| 9 AM | <p>Qualitative Graphs (from the Mathematics Teacher)</p> <p><i>Goal:</i> Participants will practice applying the graphical representations of velocity, displacement and acceleration.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants complete the Qualitative Graphs worksheet, describing physical situations for given graphs. <p>Lim debriefs the participants' results.</p> |
| 10 AM | <p>Intro to the theory of g and free fall motion equations</p> <p><i>Goal:</i> Introduce participants to the basics of free fall motion under constant acceleration due to gravity. Participants learn four basic equations for free fall motion. Participants practice the algebraic techniques of solving systems of equations by substitution.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants take notes, while Lim introduces theoretical data charts for displacement, velocity and acceleration of an object in free fall (if gravity is approximated as 10 m/s^2). • Participants take notes while Calahan introduces the finite differences method for developing equations to model the linear theoretical velocity data and the parabolic theoretical displacement data. This leads to the development of $v = v_0 + at$ and $s = v_0 t + \frac{1}{2}at^2$. • Participants work together in groups to use the two free fall motion equations developed so far, to derive the next two on their own. Participants first aim to use the two equations to relate v, s, and a. Participants then aim to use the first two equations to relate v, s, and t. This work should yield $v_f^2 - v_i^2 = 2as$ and $2s = (v + v_0) t$ • Lim reviews the four basic free fall motion equations |
| 12 PM | Lunch |
| 1 PM | <p>Ball Drop</p> <p><i>Goal:</i> Participants practice with concepts of displacement, velocity, and acceleration and the relationships between their graphs. Participants verify the gravitational constant as 9.8 m/s^2. Participants review using the TI 84 to fit a line to data, fit a parabola to data, and calculating slope. Participants also learn the ΔList option.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Each group drops a basket, hand, or tennis ball under a CBR in order to collect it's displacement vs. time data. Groups save this data as a group. • Participants use their displacement vs time data and the change in list option to develop a third list representing the velocity of the ball in free fall. |

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| | <ul style="list-style-type: none">• Participants graph this third list of average velocities vs. their time data on their calculators.• Participants find the average acceleration of the ball in free fall using the slope of the velocity graph, and note it's proximity to the gravitational constant of 9.8 m/s^2 |
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Homework!!!

The Vertex Form of Parabolas (Activity 3 in TI Explorations)

Goal:

Participants will learn the affects of the parameters a , h , and k on the graph of $y = a(x - h)^2 + k$.

Participants will practice using Transformation Graphing.

Tasks:

- Participants will look at a sequence of parabolas on their graphing calculator and note the patterns which arise. Participants will develop conjectures about the affects of a , h , and k on the parabola graph.

Week 1
Friday Agenda

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| 9 AM | <p>Go Over Homework on Vertex Form of a Parabola</p> <ul style="list-style-type: none"> • Calahan will summarize group findings and relate to translation, reflection, and dilation in the Cartesian plane. • Participants will ungroup their Ball Free Fall data, and fit a parabola to one complete bounce using their knowledge of the parameters and Transformation Graphing. |
| 10 AM | <p>Ball Toss</p> <p><i>Goal:</i> Participants extend the concepts of free fall motion to projectile motion. Participants review entering and graphing data, as well as using Transformation Graphing.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Two participants stand at each 1m points, marked out along a wall of the classroom. Lim tosses a ball along the wall and participants eyeball and mark the position of the ball as it passed. A video camera will record the time. • Participants measure the height of their position mark and then record the position and time the ball passes their meter mark on a class chart. • Participants enter the horizontal position, vertical position, and time data in their calculators. • Participants fit a line to the x vs. t data and a parabola in vertex form to the y vs. t data. Participants convert their vertex form into standard form. • Participants use substitution to take x(t) and y(t) and represent y(x). • Lim debriefs the process and discusses with participants the reasons behind the theoretical models $x = v_{0x} t$ and $y = v_{0y} t + \frac{1}{2}at^2$. • Participants use substitution to take these theoretical models for projectile motion and develop the form $y(x) = \frac{v_{0y}x}{v_{0x}} + 2g\left(\frac{x}{v_{0x}}\right)^2$ |
| 12 PM | Lunch |
| 1PM | <p>Monkey and Hunter Problem</p> <p><i>Goal:</i> Participants extend concepts learned in relation to projectile motion, and learn about parametric mode on the graphing calculator.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Given the monkey is at a height of 3m and the gun is 10m away, develop equations to describe the position of the monkey and the bullet (in terms of v) vs. time. • Graph these equations in Parametric mode on their graphing calculator. • Participants enter various velocities into the monkey position equation, to consider a number of questions: |

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| | <p>What happens if the horizontal speed of the monkey is increased or decreased?</p> <p>What determines how long it takes the bullet to reach the ground?</p> <p>What determines how far the bullet travels horizontally?</p> <p>What happens if the horizontal separation is increased or decreased?</p> <ul style="list-style-type: none">• Participants solve for where the monkey will be hit should the initial velocity be 14m/s. <p>Participants solve for the minimum velocity necessary to hit the monkey, considering, 1) the max time available to hit the monkey, and 2) how time is related to the horizontal velocity of the bullet.</p> |
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| 2:30 PM | Learning Loop Orientation |
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Homework!!!
Physics on the Learning Loop

Week 2
Monday Agenda

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| 9 AM | Review Learning Loop Homework |
| 9:30 AM | <p>Newton's Laws</p> <p><i>Goal:</i> Participants discover Newton's Second Law and are introduced to his first and third law. Participants are introduced to the EASYLINK and probes. Participants review fitting lines to data.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will complete a EASYLINK activity with force probes, accelerometers, and dynamics carts to collect force vs. acceleration data. • Participants will fit a line to the F vs. a data. • Lim will discuss the relationship found as $F=ma$, Newton's Second Law. • Participants will take notes as Lim presents all of Newton's Laws, with examples. <p>Work and Energy Introduced</p> <p><i>Goal:</i> Participants are introduced to the definition of work and formulas for kinetic energy and potential energy.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants take notes as Lim introduces the definition of work as force times displacement, including examples. • Participants take notes as Lim uses this definition to derive the formula for potential energy. • Participants take notes as Lim introduces the formula for kinetic energy, $\frac{1}{2}mv^2$. • Participants calculate the velocity and kinetic energy of an egg dropped from a six story building. |
| 12 PM | Lunch |
| 1PM | <p>Introduction to Exponential Growth</p> <p><i>Goal:</i> Participants learn the meaning of the parameters, a and b, in the exponential equation, $y = ab^x$, and learn about exponential growth and decay.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • M&Ms Activity: Participants will spill out 100 M&Ms and remove each one that spills out with the M&M face up. They will record the spill # and the number of remaining M&Ms. They will repeat until all the M&Ms are removed. They will enter data into calculator and use Transformation Graphing to fit an exponential curve to the data. <p>Dice Activity: Participants will roll 36 dice and remove each die that rolls a 6. They will record the roll # and the number of remaining dice. They will repeat until all dice are removed. They will enter data into calculator and use Transformation Graphing to fit an exponential curve to the data</p> |

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Week 2
Tuesday Agenda

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| 9 AM | <p>Ball Bounce <i>Goal:</i> Participants will apply their understanding of the equation, $y = ab^x$, to model the peak heights of a bouncing ball. <i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will use the CBR to gather displacement vs. time data of a bouncing ball. • Participants will find the average coefficient of restitution for their ball. • Using this coefficient of restitution, participants will develop an exponential model for the peak heights vs. bounce number. • Lim discusses the relation between the coefficient of restitution and the percent loss of potential energy in each bounce. <p>Water Balloon Launch! <i>Goal:</i> Participants apply the equations developed for projectile motion in the context of a water balloon launch, and verify their accuracy by comparing with gathered data. <i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants use spring scales and the format of the Hooke’s Law experiment to find the k value for the rubber tubing. Participants use a gm scale to measure the mass of their water balloon. • Participants launch their balloon with the tubing and broom handle slingshot at a given launch angle using a given stretch of the tubing. • Participants measure the range (x) of their balloon after launch. Participants also measure the “hang time.” • Participants calculate the work done on the tubing • Participants equate this work to the kinetic energy at launch and find the launch velocity of the balloon. • Participants use the launch angle and sine and cosine functions to find v_{0x} and v_{0y} • Participants use the equation $x = v_{0x}t$ to solve for x. • Participants compare this x with their measured range. |
| 12 PM | Lunch |
| 1PM | <p>Water Balloon Launch 2! <i>Goal:</i> Participants apply the equations developed for projectile motion in the context of a water balloon launch, and verify their accuracy by attempting to hit a target. <i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants use spring scales and Hooke’s Law to find the k value for the rubber tubing. Participants use a gm scale to measure the mass of their |



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| | <p>water balloon.</p> <ul style="list-style-type: none"> • A given stretch and angle of launce is set by the instructors. • Participants calculate the work done in stretching the tubing • Participants equate this work to the kinetic energy the balloon will have at launch and solve for v_0. • Participants use the launch angle and sine and cosine functions to find v_{0x} and v_{0y} • Participants use the equation $x = v_{0x}t$ to solve for x, which should be the range. • Participants use this range to position their water balloon slingshot so that it will hit the declared target. Participants then launch their balloon and compete to come closest to the target. |
| 2:30 PM | Introduction of MESA Day's Latest National Competition: Trebuchet |

Week 2
Wednesday Agenda

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| 9 AM | <p>Intro to Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn about the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will complete worksheet asking them to use Transformation Graphing to discover the effects of the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$. • Whole group discussion on discoveries and recording on large graph paper to be posted in room. |
| 12 PM | Lunch |
| 1PM | <p>Applications of Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn about harmonic motion and apply the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will affix a basketball on a rope to the ceiling. They will swing it and use the CBR to pick up displacement vs. time data for the makeshift pendulum. • Participants will use Transformation Graphing to develop a model for the data. |

Week 2
Thursday Agenda

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| 9 AM | <p>More Applications of Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn more about harmonic motion and sound, and practice applying the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$. Participants will learn to use the sound probe.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Lim will discuss sound as a wave, and wave propagation. • Participants will use tuning forks and sound probes to gather displacement vs time data for the vibrating membrane on the sound probe. • Participants will use Transformation Graphing to develop a model for the data. • Participants will compare frequencies for the different tuning forks, and develop frequency vs. octave # data. • Participants will develop an exponential model for the frequency vs. octave # data. • Light Probe |
| 12 PM | Lunch |
| 1PM | <p>Intro to Circuits and Exponential Growth</p> <p><i>Goal:</i> Participants review modeling exponentials, learn to use the voltage probe, and learn about basics of circuits.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Larry discusses the basic principles of circuits, including voltage, current, capacitance, and resistance. • Participants gather voltage vs. time data for a discharging RC circuit. Participants develop an exponential model for the data. |

Week 2
Friday Agenda

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| 9 AM | <ul style="list-style-type: none"> • Pendulum, periodicity • Kepler's Laws |
| 12 PM | Lunch |
| 1 PM | <p>Closing Ceremonies:</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Determine Follow-up schedule • Participants receive remaining TI equipment. • Participants receive a certificate of completion. |
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Week 2
Friday Agenda

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| 9 AM | <p>Intro to Circuits and Exponential Growth</p> <p><i>Goal:</i> Participants review modeling exponentials, learn to use the voltage probe, and learn about basics of circuits.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Larry discusses the basic principles of circuits, including voltage, current, capacitance, and resistance. • Participants gather voltage vs. time data for a discharging RC circuit. • Participants develop an exponential model for the data. |
| 12 PM | Lunch |
| 1PM | <p>Closing Ceremonies:</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Determine Follow-up schedule • Participants receive remaining TI equipment. • Participants receive a certificate of completion. |

Week 1
Monday Agenda

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| 9 AM | Welcome and Business: sign in, introductions, notifications re: attendance requirements and equipment ownership, W-9, and overview of institute |
| 10AM | <p>Celsius vs. Fahrenheit Activity</p> <p><i>Goal:</i> Participants will learn to use the EASYLINK with the temperature probe, input data, graph data, and fit lines to data using TI-Nspire. Participants will review parameters a and b in $y = ax + b$ and the conversion formula for Fahrenheit to Celsius.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants measure the temperature of several objects around the room, using the temperature probe with the EASYLINK. • Participants graph data on graph paper, and fit line of best fit with eye and ruler. They note the slope and the y-intercept. • Participants are led to enter data in lists, plot data using STAT PLOT, and finally approximate a line of best fit, using Transformation Graphing. • Group discusses meaning of the parameters, a and b in the equation, $y = ax+b$. |
| 12 PM | Lunch |
| 1 PM | <p>Hooke's Law Activity</p> <p><i>Goal:</i> Participants will review $y = ax + b$ and Hooke's Law for linear springs. Participants will also review entering and graphing data on the TI Nspire.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will record the displacement of a rubberband as the weight in a bucket it holds is increased. • Participants will enter Force vs. displacement data into their TI Nspire, and approximate a line of best fit. • Participants will review the meaning of a and b in the equation $y = ax + b$ |

Week 1
Tuesday Agenda

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| 9 AM | <p>Intro 1D Motion</p> <p><i>Goal:</i> Participants will learn definitions of displacement, distance traveled, speed, velocity, and acceleration.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Lim provides direct instruction with participants taking notes |
| 10 AM | <p>Walk This Way 1</p> <p><i>Goal:</i> Introduce participants to concepts of velocity and displacement: 1D Motion. Introduce participants to CBRanger and grouping data on the calculator. Review $y = ax+b$. Review inputting and graphing data on the TI Nspire.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants mark off 5m, a point of reference, and an initial position point. 1st: One group member starts at the initial position point and walks away at a constant speed, while another member measures their displacement with a CBR. Participants save this data by grouping. 2nd: Repeat with the walker traveling at an increased but still constant speed. Participants save this data by grouping as well. 3rd: Repeat with the walker starting at the 5m mark and traveling toward the point of reference at a constant speed. Participants save this data by grouping as well. • Participants enter data into their lists and graph all three sets of data on the same axes. • Participants approximate a line of best fit to each graph, using Transformation Graphing. • Calahan reviews the meaning of m and b in this application. |
| 12 PM | Lunch |
| 1 PM | <p>Walk this Way 2</p> <p><i>Goal:</i> Introduce participants to a graphical representation of velocity, displacement, distance traveled, and acceleration via the CBRanger.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Calahan asks participant to walk away from CBRanger and return and then discusses velocity, displacement, distance traveled, and acceleration |

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| | <p>on the trip.</p> <ul style="list-style-type: none"> • Calahan asks participant to walk away from CBR at a constant speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. • Calahan asks participant to walk away from CBR at an increasing speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. • Calahan asks participant to walk away from CBR at a decreasing speed, and discusses velocity, displacement, distance traveled, and acceleration on the trip. |
| <p>2:00 PM</p> | <p>Walk-A-Graph</p> <p><i>Goal:</i> Participants practice concepts via graphical interpretation of velocity and displacement. Participants practice using the CBRanger.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants are given a position vs. time graph and discuss the type of motion that would create that graph. They record their discussion and then one participant attempts to walk out the motion, while the other pair member checks their result with the CBRanger. • Repeat with a new given displacement, but pair members switch roles. Continue till all possibilities “walked.” |

Week 1
Wednesday Agenda

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| 9 AM | <p>Qualitative Graphs (from the Mathematics Teacher)</p> <p><i>Goal:</i> Participants will practice applying the graphical representations of velocity, displacement and acceleration.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants complete the Qualitative Graphs worksheet, describing physical situations for given graphs. <p>Lim debriefs the participants' results.</p> |
| 10 AM | <p>Intro to the theory of g and free fall motion equations</p> <p><i>Goal:</i> Introduce participants to the basics of free fall motion under constant acceleration due to gravity. Participants learn four basic equations for free fall motion. Participants practice the algebraic techniques of solving systems of equations by substitution.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants take notes, while Lim introduces theoretical data charts for displacement, velocity and acceleration of an object in free fall (if gravity is approximated as 10 m/s^2). • Participants take notes while Calahan introduces the finite differences method for developing equations to model the linear theoretical velocity data and the parabolic theoretical displacement data. This leads to the development of $v = v_0 + at$ and $s = v_0 t + \frac{1}{2}at^2$. • Participants work together in groups to use the two free fall motion equations developed so far, to derive the next two on their own. Participants first aim to use the two equations to relate v, s, and a. Participants then aim to use the first two equations to relate v, s, and t. This work should yield $v_f^2 - v_i^2 = 2as$ and $2s = (v + v_0) t$ |

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| | <ul style="list-style-type: none"> • Lim reviews the four basic free fall motion equations |
| 12 PM | Lunch |
| 1 PM | <p>Ball Drop</p> <p><i>Goal:</i> Participants practice with concepts of displacement, velocity, and acceleration and the relationships between their graphs. Participants verify the gravitational constant as 9.8 m/s^2. Participants review using the TI Nspire to fit a line to data, fit a parabola to data, and calculating slope. Participants also learn the ΔList option.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Each group drops a basket, hand, or tennis ball under a CBR in order to collect it's displacement vs. time data. Groups save this data as a group. • Participants use their displacement vs time data and the change in list option to develop a third list representing the velocity of the ball in free fall. • Participants graph this third list of average velocities vs. their time data on their calculators. • Participants find the average acceleration of the ball in free fall using the slope of the velocity graph, and note it's proximity to the gravitational constant of 9.8 m/s^2 |

Homework!!!

The Vertex Form of Parabolas (Activity 3 in TI Explorations)

Goal:

Participants will learn the affects of the parameters a , h , and k on the graph of $y = a(x - h)^2 + k$. Participants will practice using Transformation Graphing.

Tasks:

- Participants will look at a sequence of parabolas on their graphing calculator and note the patterns which arise. Participants will develop conjectures about the affects of a , h , and k on the parabola graph.

Week 1
Thursday Agenda

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| 9 AM | <p>Review Homework on Vertex Form of a Parabola</p> <ul style="list-style-type: none"> • Calahan will summarize group findings and relate to translation, reflection, and dilation in the Cartesian plane. • Participants will ungroup their Ball Free Fall data, and fit a parabola to one complete bounce using their knowledge of the parameters and Transformation Graphing. |
| 10 AM | <p>Ball Toss</p> <p><i>Goal:</i> Participants extend the concepts of free fall motion to projectile motion. Participants review entering and graphing data, as well as using Transformation Graphing.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Two participants stand at each 1m points, marked out along a wall of the classroom. Lim tosses a ball along the wall and participants eyeball and mark the position of the ball as it passed. A video camera will record the time. • Participants measure the height of their position mark and then record the position and time the ball passes their meter mark on a class chart. • Participants enter the horizontal position, vertical position, and time data in their calculators. • Participants fit a line to the x vs. t data and a parabola in vertex form to the y vs. t data. Participants convert their vertex form into standard form. |

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| | <ul style="list-style-type: none"> • Participants use substitution to take $x(t)$ and $y(t)$ and represent $y(x)$. • Lim debriefs the process and discusses with participants the reasons behind the theoretical models $x = v_{0x} t$ and $y = v_{0y} t + \frac{1}{2}at^2$. • Participants use substitution to take these theoretical models for projectile motion and develop the form $y(x) = \frac{v_{0y}x}{v_{0x}} + 2g\left(\frac{x}{v_{0x}}\right)^2$ |
| 12 PM | Lunch |

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| 1PM | <p>Monkey and Hunter Problem</p> <p><i>Goal:</i> Participants extend concepts learned in relation to projectile motion, and learn about parametric mode on the graphing calculator.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Given the monkey is at a height of 3m and the gun is 10m away, develop equations to describe the position of the monkey and the bullet (in terms of v) vs. time. • Graph these equations in Parametric mode on their graphing calculator. • Participants enter various velocities into the monkey position equation, to consider a number of questions: What happens if the horizontal speed of the monkey is increased or decreased? What determines how long it takes the bullet to reach the ground? What determines how far the bullet travels horizontally? What happens if the horizontal separation is increased or decreased? • Participants solve for where the monkey will be hit should the initial velocity be 14m/s. • Participants solve for the minimum velocity necessary to hit the monkey, considering, 1) the max time available to hit the monkey, and 2) how time is related to the horizontal velocity of the bullet. |
| 2:30 PM | Learning Loop Orientation |

Homework!!!
Physics on the Learning Loop



Week 2
Monday Agenda

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|---------|---|
| 9 AM | Review Learning Loop Homework |
| 9:30 AM | <p>Newton's Laws</p> <p><i>Goal:</i> Participants discover Newton's Second Law and are introduced to his first and third law. Participants are introduced to the EASYLINK and probes. Participants review fitting lines to data.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will complete a EASYLINK activity with force probes, accelerometers, and dynamics carts to collect force vs. acceleration data. • Participants will fit a line to the F vs. a data. • Lim will discuss the relationship found as $F=ma$, Newton's Second Law. • Participants will take notes as Lim presents all of Newton's Laws, with examples. <p>Work and Energy Introduced</p> <p><i>Goal:</i> Participants are introduced to the definition of work and formulas for kinetic energy and potential energy.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants take notes as Lim introduces the definition of work as force times displacement, including examples. • Participants take notes as Lim uses this definition to derive the formula for potential energy. • Participants take notes as Lim introduces the formula for kinetic energy, $\frac{1}{2}mv^2$. • Participants calculate the velocity and kinetic energy of an egg dropped from a six story building. |
| 12 PM | Lunch |
| 1PM | <p>Introduction to Exponential Growth</p> <p><i>Goal:</i> Participants learn the meaning of the parameters, a and b, in the exponential equation, $y = ab^x$, and learn about exponential growth and decay.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • M&Ms Activity: Participants will spill out 100 M&Ms and remove each one that spills out with the M&M face up. They will record the spill # and the number of remaining M&Ms. They will repeat until all the M&Ms are removed. They will enter data into calculator and use Transformation Graphing to fit an exponential curve to the data. • Dice Activity: Participants will roll 36 dice and remove each die that rolls |

| | |
|--|---|
| | a 6. They will record the roll # and the number of remaining dice. They will repeat until all dice are removed. They will enter data into calculator and use Transformation Graphing to fit an exponential curve to the data. |
|--|---|

Week 2
Tuesday Agenda

| | |
|---------|---|
| 9 AM | <p>Ball Bounce</p> <p><i>Goal:</i> Participants will apply their understanding of the equation, $y = ab^x$, to model the peak heights of a bouncing ball.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will use the CBR to gather displacement vs. time data of a bouncing ball. • Participants will find the average coefficient of restitution for their ball. • Using this coefficient of restitution, participants will develop an exponential model for the peak heights vs. bounce number. • Lim discusses the relation between the coefficient of restitution and the percent loss of potential energy in each bounce. |
| 10:30AM | <p>Water Balloon Launch!</p> <p><i>Goal:</i> Participants apply the equations developed for projectile motion in the context of a water balloon launch, and verify their accuracy by comparing with gathered data.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants use spring scales and the format of the Hooke's Law experiment to find the k value for the rubber tubing. Participants use a gm scale to measure the mass of their water balloon. • Participants launch their balloon with the tubing and broom handle slingshot at a given launch angle using a given stretch of the tubing. • Participants measure the range (x) of their balloon after launch. Participants also measure the "hang time." • Participants calculate the work done on the tubing • Participants equate this work to the kinetic energy at launch and find the launch velocity of the balloon. • Participants use the launch angle and sine and cosine functions to find v_{0x} and v_{0y} • Participants use the equation $x = v_{0x}t$ to solve for x. • Participants compare this x with their measured range. |
| 12 PM | Lunch |
| 1PM | Water Balloon Launch 2! |

| | |
|---------|--|
| | <p><i>Goal:</i> Participants apply the equations developed for projectile motion in the context of a water balloon launch, and verify their accuracy by attempting to hit a target.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants use spring scales and Hooke’s Law to find the k value for the rubber tubing. Participants use a gm scale to measure the mass of their water balloon. • A given stretch and angle of launch is set by the instructors. • Participants calculate the work done in stretching the tubing • Participants equate this work to the kinetic energy the balloon will have at launch and solve for v_0. • Participants use the launch angle and sine and cosine functions to find v_{0x} and v_{0y} • Participants use the equation $x = v_{0x}t$ to solve for x, which should be the range. • Participants use this range to position their water balloon slingshot so that it will hit the declared target. Participants then launch their balloon and compete to come closest to the target. |
| 2:30 PM | Introduction of MESA Day’s Latest National Competition: Trebuchet |

Week 2
Wednesday Agenda

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|-------|--|
| 9 AM | <p>Intro to Circuits and Exponential Growth</p> <p><i>Goal:</i> Participants review modeling exponentials, learn to use the voltage probe, and learn about basics of circuits.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Larry discusses the basic principles of circuits, including voltage, current, capacitance, and resistance. • Participants gather voltage vs. time data for a discharging RC circuit. Participants develop an exponential model for the data. |
| 12 PM | Lunch |
| 1 PM | <p>Intro to Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn about the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will complete worksheet asking them to use Transformation Graphing to discover the effects of the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$. • Whole group discussion on discoveries and recording on large graph paper to be posted in room. |

Week 2
Thursday Agenda

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|-------|---|
| 9 AM | <p>Applications of Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn about harmonic motion and apply the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will affix a basketball on a rope to the ceiling. They will swing it and use the CBR to pick up displacement vs. time data for the makeshift pendulum. • Participants will use Transformation Graphing to develop a model for the data. |
| 11 AM | <p>More Applications of Trigonometric Functions</p> <p><i>Goal:</i> Participants will learn more about harmonic motion and sound, and practice applying the parameters a, b, c and d in the equation $y = a \cos b(t - c) + d$. Participants will learn to use the sound probe.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Lim will discuss sound as a wave, and wave propagation. |
| 12 PM | Lunch |
| 1 PM | <p>More Applications continued...</p> <ul style="list-style-type: none"> • Participants will use tuning forks and sound probes to gather displacement vs time data for the vibrating membrane on the sound probe. • Participants will use Transformation Graphing to develop a model for the data. |

Week 2
Friday Agenda

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| 9 AM | <p>More Applications of Exponential Functions</p> <p><i>Goal:</i> Participants review modeling data with exponential functions, learn about the relationship between frequency and octave.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will compare frequencies for the different tuning forks, and develop frequency vs. octave # data. • Participants will develop an exponential model for the frequency vs. octave # data. |
| 10:30 AM | <p>Power Functions</p> <p><i>Goal:</i> Participants will learn the effect of the parameters, a and b, on the graph of a power function given by the equation $y = ax^b$. Participants will learn about applications of power functions.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Participants will develop a power function model for the frequency of a pendulum vs. it's length. • Participants will develop a power function model for the period of revolution of a planet vs. it's average distance from the sun. (Kepler's Law) |
| 12 PM | Lunch |
| 1 PM | <p>Closing Ceremonies:</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Determine Follow-up schedule • Participants receive remaining TI equipment. • Participants receive a certificate of completion. |

Week 2
Friday Agenda

| | |
|-------|--|
| 9 AM | <p>Intro to Circuits and Exponential Growth</p> <p><i>Goal:</i> Participants review modeling exponentials, learn to use the voltage probe, and learn about basics of circuits.</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Larry discusses the basic principles of circuits, including voltage, current, capacitance, and resistance. • Participants gather voltage vs. time data for a discharging RC circuit. • Participants develop an exponential model for the data. |
| 12 PM | Lunch |
| 1PM | <p>Closing Ceremonies:</p> <p><i>Tasks:</i></p> <ul style="list-style-type: none"> • Determine Follow-up schedule • Participants receive remaining TI equipment. • Participants receive a certificate of completion. |

Ball Bounce

In this activity, you will apply what you have learned about exponential decay to determine the way in which the peak heights of a bouncing ball vary as it bounces.

1. To do this, connect your TI 84 Plus to a CBR2. This should activate the Easy Data application on the calculator. In the Easy Data program, choose Setup, TimeGraph, and Edit to set the calculator to gather data at .05 second time intervals for 75 samples. With one group member holding the ball, one holding the CBR2 at least 0.5 meters above the ball, and one holding the calculator, have the group member holding the calculator press Start at the moment the ball is dropped.
2. When you are done collecting the data, transfer the data in L1 and L2 to each group member's calculator.
3. Each group member should now create a **STATPLOT** of L1 and L2, choose a reasonable **WINDOW** and then **GRAPH** their data.
4. Use the blue **TRACE** key to approximate the peak height of the ball at successive bounces. Record the height, h , of each bounce number, n , in the chart below:

| Bounce #, n | Peak Height, h | |
|---------------|------------------|---------------|
| 0 | | h_{n+1}/h_n |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

5. Next, you will find how much, on average, the ball rebounds after each bounce. To do this, compute h_1/h_0 , h_2/h_1 , etc. Enter this data into the third column of the chart on the first page.

What is the average of these ratios? _____

Note: $\sqrt{\frac{h_{n+1}}{h_n}}$ this known as the coefficient of restitution

6. Using your knowledge of exponential functions, write an exponential model for the Peak Height, h , in terms of the Bounce Number, n .

$$h(n) = \underline{\hspace{10em}}$$

7. Enter the Bounce Number data in L3 and the Peak Height data in L4. Create a **STATPLOT** of L4 vs. L3 (L3 in Xlist and L4 in YList), choose a reasonable **WINDOW** and then **GRAPH** the data.
8. Use the **Y=** key to access Y_1 . Enter $h(n)$ into Y_1 . Then **GRAPH** to see your model graphed with the data. How does the model compare?
9. What is the meaning of the base of your model in the context of the ball bounce?
10. What is the range of your model? Does it accurately predict how the bouncing ball will behave in the long run?
11. In introductory physics, we learn that the ball has only potential energy (the energy of position) at the top of each bounce, and only kinetic energy (the energy of motion) at the bottom of each bounce. The ball loses some energy to the environment each time it strikes the ground. This fact is evidenced by the consecutively smaller rebound heights. The potential energy of the ball at the top of each bounce is given by mgh , where m is the mass, g is 9.8 m/s^2 , and h is the height of the bounce. Since m and g are constants, the decay of the potential energy is determined by the decay in the height. Using these facts, approximate the percent of the ball's energy lost in each bounce.

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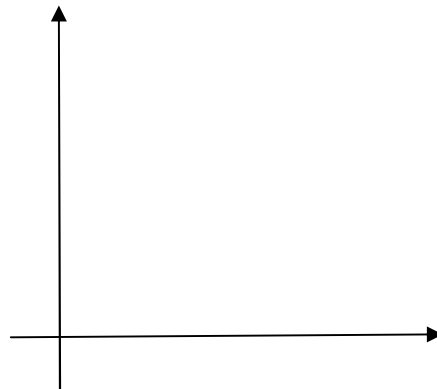


The Ball Drop

In this activity, you will test out the theories of free fall and review the meaning of slopes in the context of motion. You will also practice using the CBR2 and the Δ List option on the TI-84 Plus calculator.

1. To do this, connect your calculator to a CBR2. Use Setup and Time Graph to ready the CBR2 to collect the data. Think about how long and how quickly you'll need to collect the data given the height you plan to drop the ball from. With one group member holding the ball, one holding the CBR2 at least 0.5 meters above the ball, and one holding the calculator, have the group member holding the calculator press Start at the moment the ball is dropped.
2. When you are done collecting the data, transfer the data in L1 and L2 to each group member's calculator.
3. Graph the position vs. time data on your calculator, using **STATPLOT** with option, Plot 1, plotting L1 and L2. You will now select a portion of the data that reflects one of the ball's falls. To do this, choose 2^{nd} **Stat**, OPS, Select(. Select(will appear on the main screen. Then type 2^{nd} **1**, 2^{nd} **2**, to indicate that you want to select data from List 1 and List 2. The cursor will appear on the graph of the data. Move the cursor to the leftmost data point you want to keep and press Enter. Then move the cursor to the rightmost data point you want to keep and press Enter. Only the data you selected will now show on the graph. If you'd like to reset the window, choosing **ZOOM**, ZoomStat is the fastest way.

Sketch the data from the selected fall on the axes provided below. Be sure to label and scale your axes. **Note: Every time you graph, you may have to turn off extra, undesired equations in the $Y=$ menu or extra, undesired data plots in the **STATPLOT** menu.**

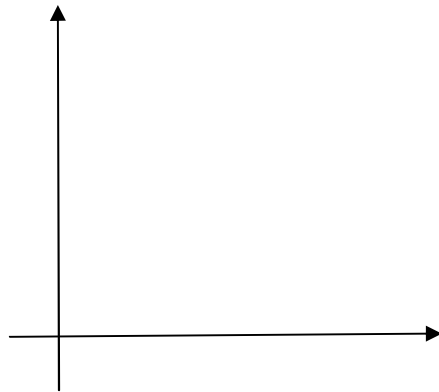


- Now each group member is going to create a list of average velocity data for the ball in free fall. Since average velocity on an interval is the change in distance over the change in time for that interval, you will be using the Δ List option to create your list of velocities.

To do this, press **STAT**, choose EDIT and move your cursor to the top of L3. Press **2nd** **LIST** and move your cursor to the right to access OPS. Then choose Δ List(. When you press **ENTER**, Δ List will appear at the bottom of the screen, where you can type in an equation to define a list. Next type L2). Access Δ List again, choose it, and type L1) to complete defining L3 as Δ List(L2)/ Δ List(L1). Choose **ENTER**. This will make L3 into a list of average velocities of the ball.

- In order to see the graph of these velocities vs. time (to graph L3 vs. L1) we have to ensure that the two lists are the same length. Otherwise, the calculator will give an error message, DimMismatch. To do this, delete the last data entry from L1.
- Graph the plot of L3 vs. L1, using **STATPLOT** with option, Plot 1, with Xlist: L1 and Ylist: L3. Sketch what you see below. Be sure to label and scale your axes.

Note: If your calculator says ERROR:Dim Mismatch, it is because the lists you are attempting to graph are different lengths. Perhaps you forgot to complete #5.

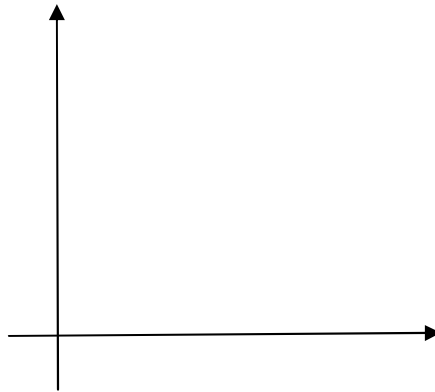


- Now each group member is going to create a list of average acceleration data for the ball in free fall. Since average acceleration on an interval is the change in *velocity* over the change in time for that interval, you will be using the Δ List option again.

To do this, press **STAT**, choose EDIT and move your cursor to the top of L4. Access Δ List, and type L3). Access Δ List again, and type L1) to complete defining L4 as Δ List(L3)/ Δ List(L1). This will make L4 into a list of average acceleration of the ball.

8. In order to see the graph of the acceleration data vs. time (to graph L4 vs. L1) we will again have to ensure that the two lists are the same length. To do this, delete the last data entry from L1.

9. Graph the plot of L4 vs. L1, using **STATPLOT** with option, Plot 1, with Xlist: L1 and Ylist: L4. Sketch what you see below. Be sure to label and scale your axes.
Note: It is normal if the graph jumps around.



10. Now, you will compare your acceleration data to the expected result. To do this, find the mean of the acceleration data in L4 by pressing **2nd LIST**, moving your cursor to the right to choose MATH, and finally choosing mean(. Next type L4, so that the calculator will find the mean (or average) of L4. Press **ENTER** and record the mean below:

Average of acceleration data: _____
Be sure to include units!

How does this average compare with the expected result?

11. Save this free fall data by grouping, so that you can use it again tomorrow. Choose **2nd MEM**, Group, Create New, and then type in BALL DROP and **ENTER**. Choose All – and scroll down to L1. Press **ENTER** when your cursor is beside L1 to choose it to be part of the group, and then repeat for L2, L3, L4 and WINDOW. Then move your cursor over to the right and choose DONE. Now the data from this experiment will be saved in a file on your calculator for you to access later.

The Ball Toss

In this activity, you will investigate parametric equations in the context of projectile motion. You have already looked at how the vertical position of an object in free fall varies with time, and now you will look at how the horizontal position of such an object varies with time as well. Finally, you will investigate the path of a ball in free fall.

1. To do this, your instructor will ask several participants to line up at 1 meter intervals. They will then throw a tennis ball past the participants. Each participant will mark where the ball passes them on the wall in front of them, and then measure the height at which the ball passed them. A video camera will be utilized to note the time corresponding to the participants' collected data. Record the resulting data in the chart below.

| time, t | horizontal position, x | vertical position, y |
|-----------|--------------------------|------------------------|
| | 0 | |
| | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| | 11 | |
| | 12 | |

2. Enter the data in your TI-84 Plus graphing calculator. Enter the time data in List 1, the horizontal position data in List 2, and the vertical position data into List 3.

3. Now, you will investigate how the horizontal position of the tennis ball varied with time. To do this, use **STATPLOT** with Option 1: Plot 1 to create a graph of L2 v. L1.

4. Make sure Transformation Graphing is turned on. In the $\boxed{Y=}$ menu, enter the equation $y_1 = ax+b$ and use Transformation Graphing to play with the values of a and b till you have the best fit you can find. Compare your choices with your group members' choices. Settle on your final choices, and enter them below:

a = _____ b = _____

$x(t) =$ _____

Note: the equation is for $x(t)$, since x was on the y – axis, and t was on the x – axis.

5. According to your $x(t)$ equation, what was the initial horizontal velocity of the ball? What was the initial horizontal position of the ball?

6. Now, you will investigate how the vertical position of the tennis ball varied with time. To do this, Use **STATPLOT** with Option 1: Plot 1 to create a graph of L3 v. L1.

7. In the $\boxed{Y=}$ menu, enter the equation $y_1 = a(x - b)^2 + c$ and use Transformation Graphing to play with the values of a , b and c till you have the best fit you can find. Note: If you recall the homework, you can determine some good guesses for b and c , by simply perusing your data chart. Compare your choices with your group members' choices. Settle on your final choices, and enter them below:

a = _____ b = _____ c = _____

$y(t) =$ _____

8. Transform your $y(t)$ equation into $y = at^2 + dt + e$ form. Remembering the lesson on free fall equations, according to your transformed $y(t)$ equation, what was the initial vertical velocity of the ball? What was the initial vertical position of the ball?

9. Solve for the zeros of $y(t)$ algebraically. What do these zeros physically represent?

10. Finally, you will investigate the path of the ball in free fall. The path of the ball is the graph of the vertical position vs. the horizontal position, or y vs. x . To see this, use **STATPLOT** with Option 1: Plot 1 to create a graph of $L3$ v. $L2$.

11. To find a model for this graph, use your equation for $x(t)$ and your equation for $y(t)$ to find an equation for $y(x)$. To do this, you will need to combine the two equations, getting rid of t .

$$y(x) = \underline{\hspace{10em}}$$

12. Enter your equation for $y(x)$ in the Y= menu and graph. How does the model fit your y vs. x data?

13. Solve for the zeros of $y(x)$ algebraically. What do these zeros physically represent?



Fahrenheit vs. Celsius

1. In this activity, you will use your calculator attached to a temperature probe to collect temperature data. To do this, first connect your TI-84Plus calculator to the Easy Link, and attach the Easy Link to the temperature probe. This should activate the calculator's Easy Data application. In this application, choose Setup, Temp, Units, and F, to ready the probe to measure temperature in degrees Fahrenheit. Choose OK, and the calculator's main screen should show the temperature of the medium surrounding the probe in Fahrenheit.

2. Measure and record the temperature in degrees Fahrenheit of a medium in the room. This could be cold water, hot coffee, room temperature air, etc. Then change the units of measurement to Celsius and repeat. Measure both Fahrenheit and Celsius temperatures for three other mediums in the room.

3. Record the class data in the table below:

| | | | | | | | | | | | | |
|---------------|--|--|--|--|--|--|--|--|--|--|--|--|
| Object | | | | | | | | | | | | |
| Temp in °F | | | | | | | | | | | | |
| Temp in °C | | | | | | | | | | | | |

4. On graph paper, create a graph of Celsius vs. Fahrenheit (this means Celsius on the y-axis and Fahrenheit on the x-axis). Use a ruler to sketch your estimate of the line of best fit. Record the slope and the x and y intercepts of your line. You may have to extend your line to see this. (Remember: The y-intercept is the y-coordinate of the point where the line crosses the y-axis. The x-intercept is the x-coordinate of the point where the line crosses the x-axis. The slope is the ratio of the change in the y-coordinate to a given change in the x-coordinate.)

Slope: _____ x-intercept: _____ y-intercept: _____

5. Your next step is to enter the class' temperature data into your calculator and produce a graph of the data. This will require entering the data into lists, setting up a scatter plot of the data in the lists, setting up a "window" appropriate for viewing the scatter plot, and finally graphing.
- To enter the data into the lists, press **STAT**. Choose Edit. Move your cursor to the first entry in List 1, and type in the first Fahrenheit value from your chart above. Continue entering the Fahrenheit data into List 1. To enter the Celsius data, move your cursor to the first entry in List 2, and type in the corresponding Celsius values from your chart above.
 - To setup a plot of your data, press **2nd STATPLOT** and choose Plot 1. Inside the Plot 1 menu, choose On, Type: scatter plot, X List: L1, YList: L2, and Mark: . (Note: if your calculator does not already have L1 chosen for the X List, choose L1 by moving your cursor next to XList and pressing **2nd 1**.)
 - To setup an appropriate "window", press **WINDOW**, and enter X min: -10. (Note: the negative sign is beneath the **3** – do not use the subtract sign.) Enter Xmax: 100, Xscl: 10, Ymin: -10, Ymax: 100, and Yscl: 10.
 - To graph the data, press **GRAPH**.
6. To fit a line to the data on your calculator, you will use an Application called Transformation Graphing. Press **APPS** and choose Transfrm. Press **Y=** and make sure your cursor is after the $Y_1=$. Enter the general equation of a line in slope intercept form, $ax + b$. (Note: you can type letters by pressing the **ALPHA** key followed by the key with the desired green letter above it. You can type the variable, x , by pressing the **X, T, θ , n** key.) Finally, press **GRAPH** and your data plot along with values for a and b should appear. Move your cursor to the $a=$ or $b=$ and type numeric values of your choice, until you feel that you have a line of good fit.
7. Record your line of good fit below. How do the a and b compare to the slope and y-intercept you calculated with your hand plotted graph?

$$y = a x + b$$

$$= \underline{\quad} x + \underline{\quad}$$

8. What does a have to do with the graph of the line? What is the meaning of a in the physical context of the experiment?

9. What does b have to do with the graph of the line? What is the meaning of b in the physical context of the experiment?

10. You may remember that there is a conversion formula relating Fahrenheit to Celsius temperatures. This conversion formula is actually the theoretical linear model for the data you have graphed. Derive this formula, in the form $C = aF + b$ (like $y = ax + b$). Use the fact that the points (32 °F, 0°C) and (212 °F, 100°C) are on the line to solve for a and b .

Force vs. Displacement

1. In this activity, you will drop nine Hershey’s kisses at a time into a bucket hanging from a rubber band. You will measure the displacement of the rubber band as the force applied to it by the Hershey’s kisses increases.

In this context, displacement is the total amount the rubber band has extended from its length when only the empty bucket is hanging from it. To help you calculate displacement, you will need to measure the length when only the empty bucket is hanging from the rubber band. This is called the “resting” length.

In this context, the force applied to the rubber band by the Hershey’s kisses is the product of their mass and the acceleration due to gravity. The mass of nine Hershey’s kisses is 41g. So, the force 9 Hershey’s kisses apply to the rubber band

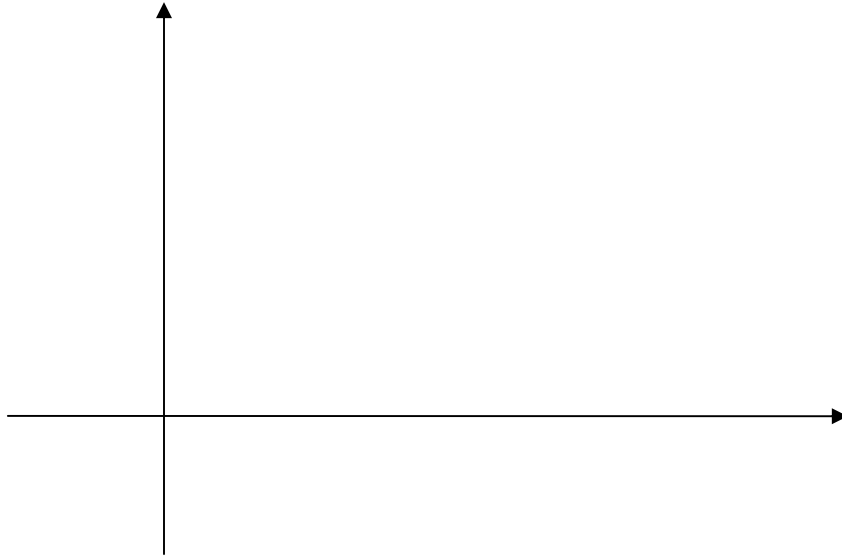
is $(41g) \left(9.8 \frac{m}{s^2} \right) \approx 0.402 \frac{kg \cdot m}{s^2}$. (Note: $\frac{kg \cdot m}{s^2}$ are called *Newtons*).

Carry out the experiment, and record your data in the chart below.

| length of rubber band | total stretch (displacement), in cm | displacement, <i>x</i> , in meters | # of groups of Hershey’s kisses | force, <i>F</i> , in Newtons (# of groups of kisses · 0.402 Newtons) |
|-----------------------|-------------------------------------|------------------------------------|---------------------------------|--|
| | 0 | 0 | 0 | 0 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

2. Enter the force vs. displacement in meters data into your calculator, using **STAT** Edit. Enter the displacement in meters data into L1 and the force data into L2. Note: To clear out a list of old data, move your cursor to the very top of the list, press **CLEAR** and **ENTER**.
3. Setup a **STATPLOT** to graph the data.

4. Setup an appropriate **WINDOW** for the data. Refer to the data to determine what an appropriate window might be.
5. Turn off any equations which are “turned on” in **Y=**. Do this by pressing **Y=**, moving your cursor on top of any equal sign that is darkened (highlighted), and finally pressing **ENTER** to “un-highlight” the equal sign.
6. Press **GRAPH** to graph the data and sketch what you see below. Be sure to indicate scale on your axes.



7. Use Transformation Graphing to find a line of good fit for your data. Record the best fit you found below:

$$F = \underline{a}x + \underline{b}$$

$$= \underline{\quad}x + \underline{\quad}$$

8. What is the meaning of a in the physical context of the experiment?
9. What is the meaning of b in the physical context of the experiment?

FYI: Hooke’s Law says that the force, F , exerted by a spring (the rubberband) is given by $F = kx$, where x is the displacement the force causes and k is a constant associated with the individual spring.





Exponential Functions and Music

1. Your instructor will give you a tuning fork and a microphone probe. Use the Data Mate application to access the Setup menu and choose Mode, time graph. Have your partner strike the tuning fork against the edge of your shoe and then hold the tuning fork so that its base is touching the desk. Quickly move the microphone probe near to the tuning fork, and choose Start on the calculator. When data collection is finished, you will see a graph of the vibration of the membrane in the microphone probe vs. time. Use your right arrow to trace along the data, and record the x coordinates of two consecutive peaks.

$$t_1 = \underline{\hspace{2cm}} \qquad t_2 = \underline{\hspace{2cm}}$$

2. Use these two values to compute the period, T , for your tuning fork's note. Remember, period is the number of seconds it takes for one cycle to complete.

$$T \text{ for note } \underline{\hspace{2cm}} \approx \underline{\hspace{2cm}}$$

(C, D, E, etc.)

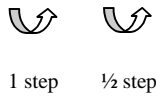
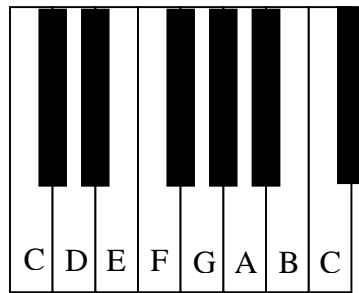
3. The more octaves a note is above middle C, the higher the frequency of the note. In other words, the frequency and the number of octaves it is above middle C are related. The frequency, f , is the number of cycles completed per second and it is given by $f = 1/T$. Use this definition for frequency and your period measurement from #2 to find the frequency of your note.

$$f \text{ for note } \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

(C, D, E, etc.)

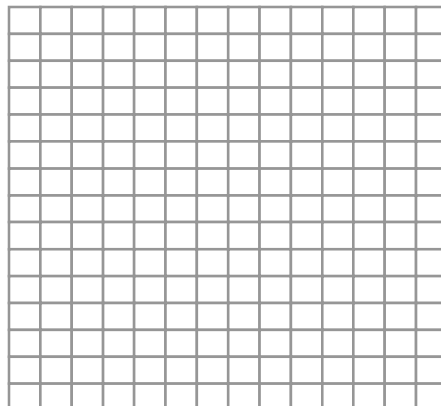
4. Record the class data your instructor collects for each note in the octave above middle C in the chart below.

| number of octaves above middle C | Note in the octave above middle C | F , in cycles/sec or Hertz |
|----------------------------------|-----------------------------------|------------------------------|
| | C | |
| | D | |
| | E | |
| | F | |
| | G | |
| | A | |
| | B | |
| | C | |



1 step 1/2 step

5. Set up and graph a scatterplot of the frequency vs. the number of octaves above middle C. Be sure to enter the number of octaves above middle C in List 1 and the frequency in List 2. What type of function (linear, quadratic, exponential, trigonometric, etc.) might provide a reasonable model for the data? Sketch what you see on the graph provided below:



6. Activate the APP, Transformation Graphing. Using the $y=$ menu, enter $y_1 = ab^x$. Press **GRAPH** and vary the values of a and b until you achieve what you believe is a good fit for the data. Record your good fit here:

$$f(n) = \underline{\hspace{10em}}$$

7. In *Calculus*, by Hughes-Hallet and Gleason, the authors state that the theoretical model for this set of data is $f(n) = 256(2)^n$. How does this compare to your good fit model?

The Family of Sinusoidal Functions

In this activity, you will learn the affect that the parameters, a , b , c , and d have on the graph of the function $y = a \cos(b(x - c)) + d$. You will also learn more about using Transformation Graphing.

1. On your TI 84 Plus, press **MODE** and choose Rad (for radians). Press **Y=** and enter $y = a \cos(b(x - c)) + d$ into y_1 . Next, press **ZOOM** and choose option 7: ZoomTrig. Finally, press **GRAPH**. Enter values for the parameters as follows:
 $a = 1$ $b = 1$ $c = 0$ $d = 0$

2. Vary the value of a by either moving your cursor next to $a =$ and typing in a value of your choice, or by pressing the right and left arrows. Answer the following questions:
 - a) What happens to the graph as you increase the value of a beyond 1?

 - b) What happens to the graph when you decrease the value of a below 1, while keeping it greater than 0?

 - c) What happens to the graph when you choose negative values of a ?

3. Vary the value of c by either moving your cursor next to $c =$ and typing in a value of your choice, or by pressing the right and left arrows. Answer the following questions:
 - a) What happens to the graph as you increase the value of c beyond 0?

 - b) What happens to the graph when you decrease the value of c below 0?

4. Vary the value of d by either moving your cursor next to $d =$ and typing in a value of your choice, or by pressing the right and left arrows. Answer the following questions:
- a) What happens to the graph as you increase the value of d beyond 0?

 - b) What happens to the graph when you decrease the value of d below 0?
5. Vary the value of b by either moving your cursor next to $b =$ and typing in a value of your choice, or by pressing the right and left arrows. Answer the following questions:
- a) What happens to the graph as you increase the value of b beyond 1?

 - b) What happens to the graph when you decrease the value of b below 1, while keeping it greater than 0?

 - c) Choose $b = 3$, and record the period of the graph. Note: the period is how many x units it takes the graph to complete one cycle. Also, it will help to know that the marks along the x axis are every $\frac{\pi}{2}$.

- d) Repeat c) for the following b values, and record the results in the chart below:

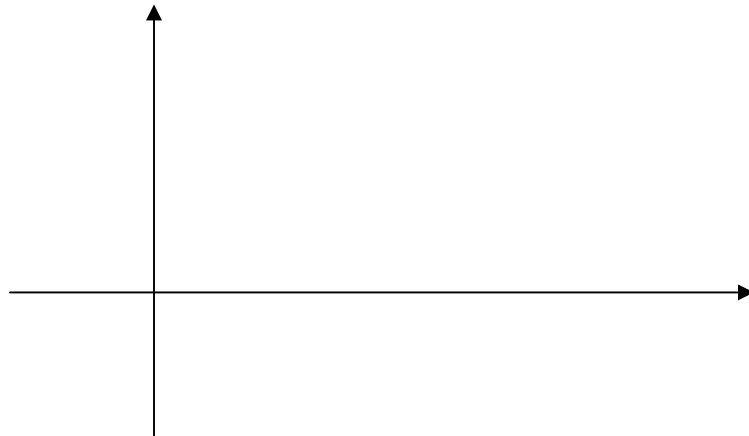
| b | Period, T |
|---------------|-----------|
| $\frac{1}{4}$ | |
| $\frac{1}{2}$ | |
| 1 | |
| 2 | |
| 3 | |

Note: You will have to widen your WINDOW to see a full period for $\frac{1}{4}$ and $\frac{1}{2}$.

- e) How do T and b relate? Develop an equation relating T to b and record below:

$$T = \underline{\hspace{2cm}}$$

6. Sketch what you think the graph of $y = 2 \cos 0.5(x - \frac{\pi}{2}) + 3$ would look like. Then check your work on your graphing calculator.



Introduction to Exponential Decay 1

In this activity, you will practice inputting data, graphing data, and fitting curves to data using the TI-84 Plus and Transformation Graphing. You will also investigate the meaning of the parameters in the equation of the exponential curve: $y = ab^x$

1. With your group, count out 100 pennies.
2. Toss the pennies and note all those that come up heads. Remove all the pennies that come up heads and note how many pennies are left. Repeat this process until no pennies remain or you have tossed 15 times, which ever comes first. Record the number of times you've completed the process vs. the number of pennies remaining in the chart below:

| Toss Number | Number of remaining pennies |
|-------------|-----------------------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |

3. You will now investigate what type of function can model the data you collected. To do this, let's first graph the data. Enter the data on your TI-84 Plus. Enter Toss Number, n , as L1 and Number of Remaining Pennies, P , as L2. Next, set up a **STATPLOT** of your data, choose a **WINDOW** for the data plot, and finally **GRAPH**.
4. In the **Y=** menu, enter the equation $y_1 = ab^x$. Note: you can raise to the power of x using the **^** key and then **x**. Next, use Transformation Graphing to play with the values of a and b till you have the best fit you can find. Record the final choices which produce your best fit below:

Hint 1: What is the y intercept of an equation in the form $y_1 = ab^x$? To answer this, remember the y intercept is the y value when $x = 0$. Hint 2: Considering the pennies experiment, what should the y intercept of the model be?

$a =$ _____ $b =$ _____

Consider the parameters a and b . In your group, discuss the answers to the following.

5. In terms of the pennies activity, what is the physical meaning of a ?
6. How is a related to the graph of the curve?
7. In terms of the pennies activity, what is the physical meaning of b ?
8. How is b related to the graph of the line?

9. Use the probability of tossing a head to compute how many pennies you'd expect to be remaining after each of the below toss numbers. Show your work.

| Toss Number, n | Expected number of remaining pennies, P |
|------------------|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

10. Use two data points in the above chart to compute the theoretical model (in the form $y = ab^x$) for the pennies experiment.

$$P(n) = \underline{\hspace{10em}}$$

Intro to Exponential Decay 2

In this activity, you will review the meaning of the parameters in the equation of the exponential curve: $y = ab^x$ and practice inputting data, graphing data, and fitting curves to data using the TI-84 Plus.

1. With your group, count out 36 dice.
2. Roll the dice and note all those that roll a 6. Remove all the dice that rolled a 6 and note how many dice are left. Repeat this process until no dice remain or you have rolled 15 times, which ever comes first. Record the number of times you've completed the process vs. the number of dice remaining in the chart below:

| Roll Number | Number of remaining dice |
|-------------|--------------------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |

3. Enter the data on your TI-84 Plus using the **STAT** menu with option **1:Edit...** .

4. Set up a plot of your data on your TI-84 Plus using the **STATPLOT** menu with option **1:Plot 1**. Within the **1:Plot 1** menu, remember to turn on the plot, choose the type of plot, the lists to be plotted, and the type of mark you'll use to indicate each point.
5. Choose a window for the plot of your data. You'll need to use the **WINDOW** menu, and enter a reasonable domain for the x axis and range for the y axis as well as choose a scale for each variable. Remember, you'll be able to see your graph by simply pressing **GRAPH**.
6. Finally, use the application, Trnsfrm, located under **APPS** to find a good exponential fit for your data. You'll have to enter : $y = ab^x$ into Y1 in the **Y=** menu. Then choose various values of a and b while looking at the graph, until you find a good fit. Record your good fit below:

Consider the parameters a and b value with your “curve of good fit”. In your group, discuss the answers to the following.

7. In terms of the dice activity, what is the physical meaning of a ?
8. How is a related to the graph of the curve?
9. In terms of the dice activity, what is the physical meaning of b ?
10. How is b related to the graph of the curve?

11. Use the probability of rolling a 6 to compute how many dice you'd expect to be remaining after each of the below roll numbers.

| Roll Number, t | Expected number of remaining dice, N |
|------------------|--|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

12. Use two data points in the above chart to compute the theoretical model (in the form $y = ab^x$) for the dice experiment.

$N(t) =$ _____

Kepler's Law and Power Functions

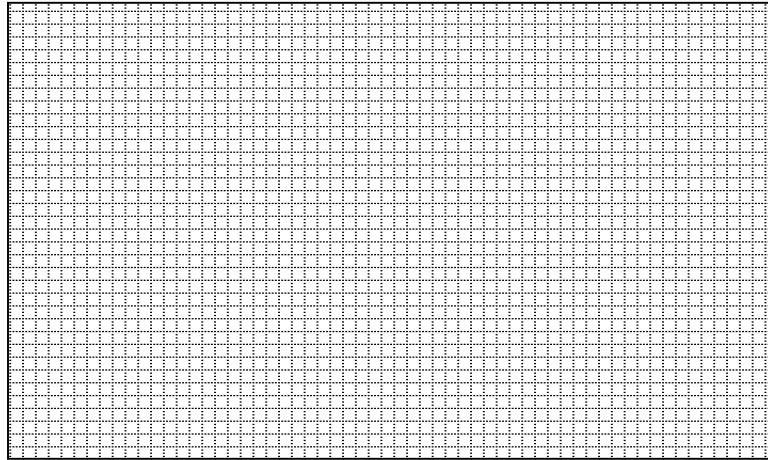
Johannes Kepler was an astronomer who used the laws of physics and gravity to describe and analyze the motion of the planets. One of his most important laws gives a relationship between the period, P , and its distance, d , from the sun. P vs. d data is provided in the table below:

| PLANETS | Average distance from the sun (in Astronomical Units) | Period of Revolution (in years) |
|---------|---|---------------------------------|
| Mercury | 0.39 | 0.241 |
| Venus | 0.72 | 0.616 |
| Earth | 1 | 1 |
| Mars | 1.52 | 1.882 |
| Jupiter | 5.20 | 11.9 |
| Saturn | 9.54 | 29.5 |
| Uranus | 19.2 | 84.0 |
| Neptune | 30.0 | 165 |
| Pluto | 39.5 | 249 |

1. Enter the data above into your calculator under **STAT** Edit, set up a **STATPLOT**, of P vs. d , set an appropriate **WINDOW**, and view the data. Describe how the period and distance from the sun are related. What type of function would model this data well? Justify your reasoning.

2. Kepler's Law states that "the square of the period of revolution for a planet is equal to the cube of its distance from the sun." Use Transformation Graphing to find a reasonable model for the data, and record your result below.

3. Graph your model with the data and record what you see below:



4. Explain algebraically how your model compares to the statement of Kepler's Law.

5. If astronomers were to discover a new planet that takes 286 years to go around the sun, how far from the sun would it be? Show your work.

Modeling Waves

In this activity you will review the general form of a sinusoidal function while using it to model a sound wave. You will strike a tuning fork beside a microphone probe connected to a CBL2 and collect data relating the displacement, D the sound wave causes on the microphone membrane vs. time, t . We are able to do this because sound is a pressure wave that travels through a medium causing displacement, which results in hearing for humans!

1. Connect a microphone probe to Channel 1 in a CBL2, and connect the CBL2 to a TI 84 Plus calculator. Use the APPS menu to choose EasyData. EasyData should recognize the microphone probe and setup to gather data every $1/10,000^{\text{th}}$ of a second for .02 seconds. Strike the tuning fork against the bottom of your shoe, hold it to the desk and press Start to gather the data.
2. Next, transfer the data to the other group members. You will need to send L1 and L2 as well as the Window. Create a plot of the data using **STAT PLOT** and then **GRAPH**. Sketch what you see in the space below. Be sure to label and scale your axes.



3. What type of graph best represents the data? Explain your reasoning.

4. a) Now you will investigate a good fit for the data. Use the general form of a sinusoidal function, $y = A \cos b(x - c) + d$ and Transformation Graphing to find values of a , b , c , and d that produce the best fit you can find. Note: Remember the meaning of the parameters, a , b , c , and d and use your data to help you do this.

b) Let $D(t)$ be this best fitting curve. Record $D(t)$ below.

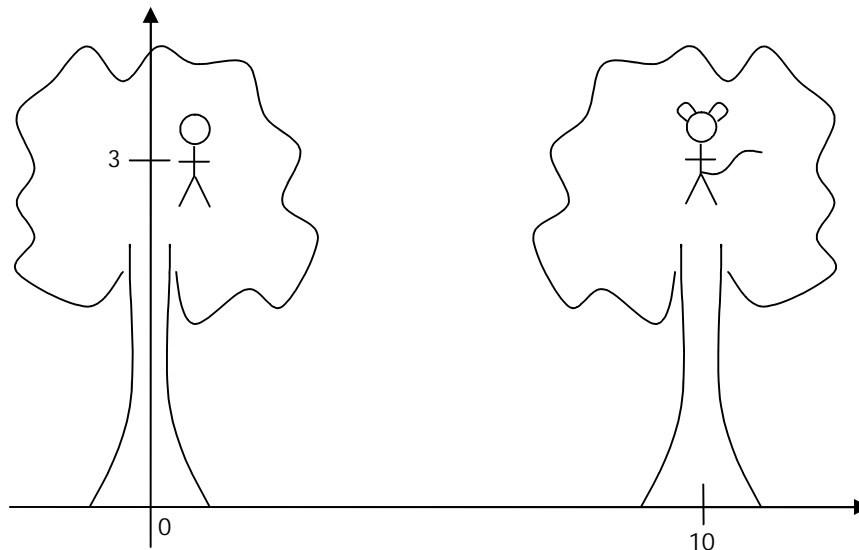
$D(t) =$ _____

The Monkey and the Hunter

In this activity, you will apply the equations you have learned for describing projectile motion. You will also learn about using parametric equations on the TI 84 Plus.

A classic problem in physics involves a hunter aiming a gun at a monkey in a tree. One question to consider is what should the monkey do? Should he stay where he is, or drop down? We will look at the position equations for both the bullet and the monkey to investigate the motion involved in this situation.

For our investigation, let's suppose that the monkey and the gun are at a height of 3m, and that the gun is a horizontal distance of 10m away from the monkey. Let's suppose also that the monkey decides to drop down.



1. Develop equations describing the horizontal and vertical position of the monkey.

$$x(t) = \underline{\hspace{4cm}} \qquad y(t) = \underline{\hspace{4cm}}$$

2. Develop equations describing the horizontal and vertical position of the bullet.
Note: leave your horizontal equation in terms of v_0 and t , where v_0 is the initial horizontal velocity.

$$x(t) = \underline{\hspace{4cm}} \qquad y(t) = \underline{\hspace{4cm}}$$

3. By graphing these equations in parametric mode on your graphing calculator, you can “animate” the monkey and the hunter. To do this, press **MODE** and choose PAR (fourth level in the menu). Next, press **Y=** and you should see places to enter equations for both x and y . Type the monkey equations you wrote in #1 into x_1 and y_1 . Next, type the bullet equations you wrote in #2 into x_2 and y_2 .

Note: Before you press **GRAPH**, you’ll have to choose a particular v_0 . So, type whatever v_0 you choose when you enter the bullet’s $x(t)$.

Move your cursor to the far left of x_1 . Press **ENTER** twice to choose the animated graphing option. Move your cursor to the far left of x_2 . Press **ENTER** four times to choose the dashed graphing option. Set a reasonable **WINDOW** press **MODE** and choose simultaneous mode, and then, finally, press **GRAPH** !

4. If the bullet has an $v_0 = 14$ m/s, when will the monkey be hit? At what height will the monkey be when he is hit?

5. a) How many seconds does it take the monkey to hit the ground?

b) Use your $x(t)$ equation and your answer in a) to figure the minimum v_0 at which the bullet must be fired in order to hit the monkey before he hits the ground.

Notes on Newton's Laws...

Law #1...

Law #2...

Law #3...

Equations of Free Fall Motion

Looking at acceleration, velocity and position in free fall:

| time, t | acceleration, a |
|-----------|-------------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

| time, t | velocity, v |
|-----------|---------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

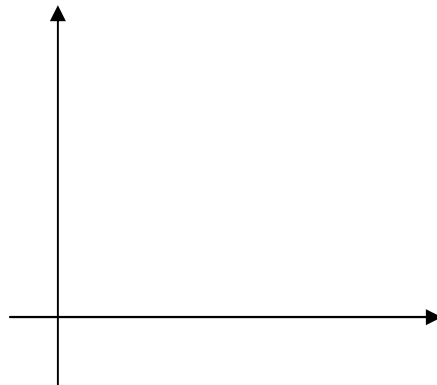
| time, t | position, s |
|-----------|---------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

Using finite differences to find equations for the data:

| time, t | velocity, v |
|-----------|---------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| t | $v(t) =$ |

| time, t | position, s |
|-----------|---------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| t | $s(t) =$ |

The graphs of $a(t)$, $v(t)$, and $s(t)$



In your group use $v(t)$ and $a(t)$ to relate v , s , and a . (In other words, combine those two equations, getting rid of t .)

Notes on Work and Energy...

Definition of Work....

Using the definition of Work to derive the formula for Potential Energy....

The formula for kinetic energy....

Calculating the velocity and kinetic energy of an egg dropped from a six story building....



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Investigating Pendulum Length and Period

1. With string, strong tape, and a ball, make a pendulum and affix it where it can swing freely. Be sure to make a pendulum of different length than other groups. Record the length of the pendulum in meters: _____

2. Connect your CBR2 to your TI 84Plus, and use the Easy Data program to set up an experiment of no more than a few seconds. Work with a partner and use the CBR2 to collect data describing the distance, d , between the CBR and the pendulum versus time, t , in seconds. In order to help the CBR2 pickup the ball, be sure not to hold the CBR too close and not to create too large a swing.

3. Once the data is collected, set up **STAT PLOT** 1 to scatter plot List1 on the x axis, and List2 on the y axis. Use **ZOOM**, **ZOOM STAT** to graph the data. Sketch one cycle of what you see in the space below. Be sure to label and scale your axes.



4. Use **TRACE** to find the period of the graph – this means find how many seconds it takes for one cycle to occur.

$T =$ _____

5. Gather data from the other groups to complete the chart below:

| Length, in meters | Period, T, in seconds |
|-------------------|-----------------------|
| | |
| | |
| | |
| | |
| | |

6. Enter the data from the chart into your lists and graph the data.
Sketch what you see in the space below. Be sure to label and scale your axes.



6. Use Transformation Graphing to develop a power function that fits the data.

$y =$ _____

7. The equation from Newton's Laws for the period vs the pendulum length is

$T = 2\pi\sqrt{\frac{l}{g}}$. Does your equation say something similar? Explain.

8. Use the equation $T = 2\pi\sqrt{\frac{l}{g}}$ to predict what happens to the period as the length of the pendulum increases without bound.

9. Use the equation to predict how much the period changes when the length changes from 1 to 2 meters. Repeat for a change in length from 5 to 6 meters. As the length increases, how does the period change?

Preparing for the Water Balloon Launch!

Materials:

Launcher
Force probe or spring scale
Meter sticks
Balance
Gram scale
Long measuring tape
Stop watches

1. Using a spring scale or a force probe, measure the force necessary to stretch the surgical tubing 0.25m. Use this measurement to approximate the "spring constant" of the surgical tubing.
2. Using the balance provided, find the mass of the balloon.
3. Setup the water balloon launcher on the field. Place the balloon in the launcher, pull the surgical tubing back so that the balloon's vertical position is 0m, measure the stretch on the surgical tubing. Also measure two lengths necessary to determine the launch angle.
4. Release the balloon and time the flight.
5. Measure the horizontal distance the balloon.
6. Using the above measurements:
 - a. Find the initial velocity in the horizontal direction.
 - b. Find the initial velocity of the launch.

- c. Find the kinetic energy of the balloon at launch.
- d. Remember, the kinetic energy from c) equals the potential energy given to the balloon by the stretch of the surgical tubing which is $\frac{1}{2}kx^2$. What does this imply the stretch of the surgical tubing was? How close is this to the actual stretch you measured?
- e. Use trigonometry and b) to find the initial velocity in the vertical direction.
- f. What does e) and the formula for range we derived in class, $R = \frac{-2v_{0x}v_{0y}}{g}$, imply the range of the balloon was? How close is this to the actual range you measured?
- g. Use trigonometry to show that $\frac{-2v_{0x}v_{0y}}{g} = -\frac{2v_0^2 \sin \theta \cos \theta}{g}$.

The Water Balloon Launch!

Materials:

Surgical Tubing
Wooden upright posts
Funnel
Fasteners of any type to hold tubing to uprights

Rules of Launch Contest:

1. Teams will calculate the k value of the surgical tubing and the mass of their balloons at the time of the contest.
2. Two water balloons will be provided to each team at the time of the contest.
3. Teams will be given the angle of launch and the distance between the launcher and the target at the time of the contest.
4. All teams will be given the same amount of time to make calculations and present their calculations on paper to the judge. They will then be given a number for their order in the competition.
5. Once their number is called, teams will have 5 minutes to fire the balloon at the set target and fire.
6. The team whose water balloon lands closest to the center of the target will be judged the winner.
7. In the event of a tie, a "launch-off" will occur in which a new target distance will be chosen.

Modeling the Motion of a Pendulum – 2D Motion

In this activity, you will apply what you learned about trigonometric functions to find a model for the position of a basketball in reference to the CBR as it swings like a pendulum.

1. With string, strong tape, and a ball, make a pendulum and affix it where it can swing freely. Record the length of the pendulum: $l =$ _____.
2. Connect a TI 84 Plus to a CBR2. This should activate the Easy Data application. Choose Setup, TimeGraph, Edit, .02, Next, 120, Next, OK. Work with a partner to use the CBR2 to collect data describing the position, s , of the swinging ball in reference to the CBR2 versus time, t , in seconds.
3. Once the data is collected, transfer the data to the other group members. You will need to send L1 and L2 as well as the Window. Create a plot of the data using **STAT PLOT** and then **GRAPH**. Sketch what you see in the space below. Be sure to label and scale your axes.



4. a) Now you will investigate a “good fit” for the data. Use the general form of a sinusoidal function, $y = a \cos b(x - c) + d$ and Transformation Graphing to find values of a , b , c , and d that produce the best fit you can find. Note: Remember the meaning of the parameters, a , b , c , and d and your data to help you do this.

Some Background on the Pendulum Activity:

The restoring force of a pendulum is given by $F = -mg \sin \theta$. By applying Newton's second law to the motion, we obtain $ma = -mg \sin \theta$. Since $a = \frac{d^2x}{dt^2}$,

$$m \frac{d^2x}{dt^2} = -mg \sin \theta.$$

For very small θ , $\sin \theta \approx \theta$, so we can write $m \frac{d^2x}{dt^2} = -mg\theta$ for small θ . Using the arc length formula, $s = r\theta$, we can write $x = l\theta$, where l is the length of the pendulum.

This implies that $m \frac{d^2x}{dt^2} = -mg \frac{x}{l}$ or $\frac{d^2x}{dt^2} = -\frac{g}{l}x$.

We now find a solution to this differential equation by inspection. This differential equation indicates that x is a function whose second derivative is the opposite of x , just "off by the constant", $-g/l$. Sines and cosines satisfy this requirement and are bounded and periodic as is pendulum motion. This leads us to suppose $x = A \cos(\omega t + \beta)$ as a

solution. Note that with this choice, $\frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \beta) = -\omega^2 x$. This indicates

that if $\omega^2 = \frac{g}{l}$, $x = A \cos(\omega t + \beta)$ is a solution of the differential equation describing the pendulum's motion.

Thanks to Millicent Anderson, retired AP Physics Instructor, Venice High School, for guidance with these ideas.



Circuits and Exponential Decay

In this activity, you will apply what you have learned about exponential decay to find a model for the discharging voltage in a capacitor. You will also learn to use a voltage probe.

Setup: Group of three, 9Volt battery, 220 μ F/35v capacitor, 100K Ohm/.25watt resistor, CBL2, voltage probe, and a TI 84 Plus.

1. Choose one of the group's TI84 Plus calculators to use. Connect your calculator to the Easy Link, and the Easy Link to the voltage probe. Choose to collect 30 samples at 1 second intervals.

Touch the 220 μ F/35v capacitor and 100K Ohm/.25watt resistor setup to the 9 Volt battery for a few moments. Place the voltage probe's red clamp onto the wire which is touching the positive terminal and its black clamp onto the other wire. Then remove the setup from the battery, and collect the data.

2. Transfer the data to the rest of the group.
3. Graph the data using the **STAT PLOT** menu. You will need to tell the STAT PLOT to graph lists TVolt and Volt, as opposed to L_1 and L_2 . You can access these new list names by pressing 2nd **LIST** and moving your cursor down until you can choose the list TVolt or Volt.
4. Consider the graph of the data. What type of function might model this data well? Explain your reasoning.
5. a) Press Y= and enter the equation $y = ab^x$. Next, use Transformation Graphing to play with the values of a and b till you have a "curve of good fit". Note: If you recall the meaning of a and the context of the experiment, you can guess a reasonable approximation for a .

Let $V(t)$ be the "curve of good fit".. Record $V(t)$.

$$V(t) = \underline{\hspace{10em}}$$

6. What is the physical meaning of the y intercept, a , of $V(t)$?

7. What is the physical meaning of the base, b , of the exponential function, $V(t)$?
8. In the study of circuits, we learn that the model for the above experiment should be of the form $V = V_0 e^{-t/RC}$ (see next page for more details). Thus the base of our exponential model for the voltage data should be close to $e^{-1/RC}$, where the C is the capacitance of $220 \mu\text{F}$ and R is the resistance of 100K Ohms . How close does your value of b come to the theoretical model?

Some Background on the Voltage Activity:

One of Kirchoff's Laws states that the voltage supplied by the battery, V_B , is equal to the sum of the voltage drops over the capacitor and the resistor. This means $V_B = V_C + V_R$ where V_C and V_R are the voltage drops over the capacitor and resistor respectively.

Ohm's Law states that the voltage drop over the resistor is given by $V_R = IR$, where I is the current through the resistor and R is its resistance. The drop over the capacitor is given by $V_C = Q/C$, where Q is the charge and C is its capacitance. Thus Kirchoff's Law mentioned above becomes $V_B = IR + Q/C$.

Since the current is the amount of charge passing through a given point per unit time, $I = dQ/dt$. The battery is removed from the circuit when discharging, so our situation was described by $0 = \frac{dQ}{dt}R + Q/C$.

We can solve this differential equation using separable differential equations as follows:

$$\begin{aligned} \frac{dQ}{dt}R = -Q/C &\Rightarrow \frac{dQ}{Q} = \frac{-1}{RC}dt \Rightarrow \int \frac{dQ}{Q} = \int \frac{-1}{RC}dt \Rightarrow \ln|Q| = \frac{-1}{RC}t + k \\ \Rightarrow Q(t) = e^{-t/RC}e^k &\Rightarrow Q(t) = Ke^{-t/RC} \end{aligned}$$

Given that $Q(0) = Q_0$, $Q(t)$ becomes $Q(t) = Q_0e^{-t/RC}$.

Now, to relate this to the voltage data we collected. Since the voltage on the capacitor, V_C , is given by $V_C = Q/C$ (as mentioned above), the charge on the capacitor, $Q(t)$, is directly proportional to V_C . This indicates that the voltage we measured with the probe will be given by an exponential equation also. Namely, since $V_C = Q/C$ and

$$Q(t) = Q_0e^{-t/RC}, \quad V_C = \frac{Q_0e^{-t/RC}}{C}.$$

Finally, since C is constant, $\frac{Q_0}{C} = V_0$. This reduces $V_C = \frac{Q_0e^{-t/RC}}{C}$ to $V_C = V_0e^{-t/RC}$.

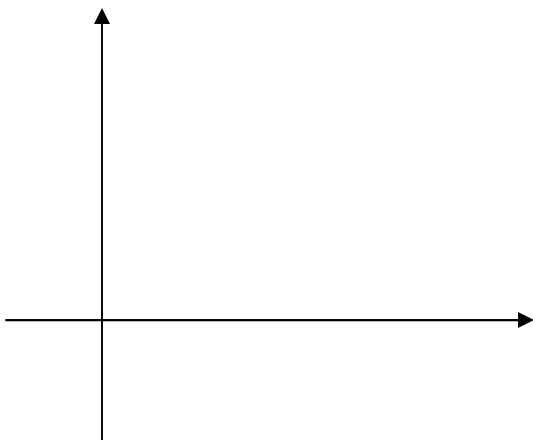
Thus the base of our exponential model for the voltage data should be close to $e^{-1/RC}$, where the capacitance is $220 \mu\text{F}$ and the resistance is 100K Ohms .

Thanks to Millicent Anderson, retired AP Physics Instructor, Venice High School, for guidance with these ideas.

1D Motion Notes

As the instructor has participants walk out various graphs and discusses the related average velocity, final displacement, distance traveled, and acceleration, please record below:

Walk away at constant speed:



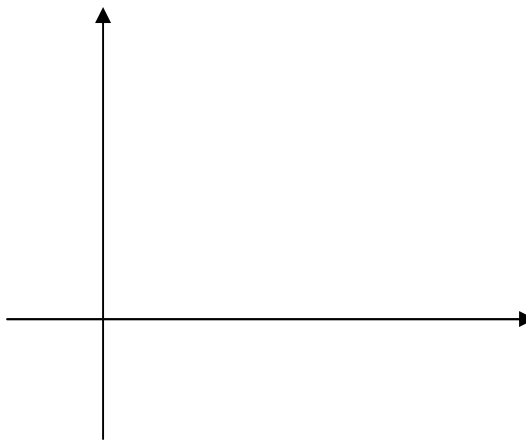
Displacement: _____

Total Distance Traveled: _____

Average Velocity: _____

Acceleration: Pos Neg Zero

Walk away at a decreasing speed:



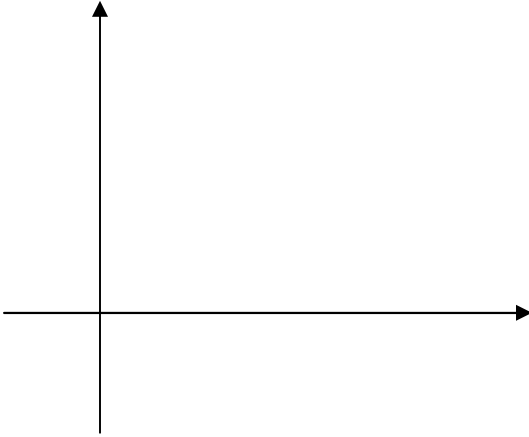
Displacement: _____

Total Distance Traveled: _____

Average Velocity: _____

Acceleration: Pos Neg Zero

Walk away at an increasing speed:



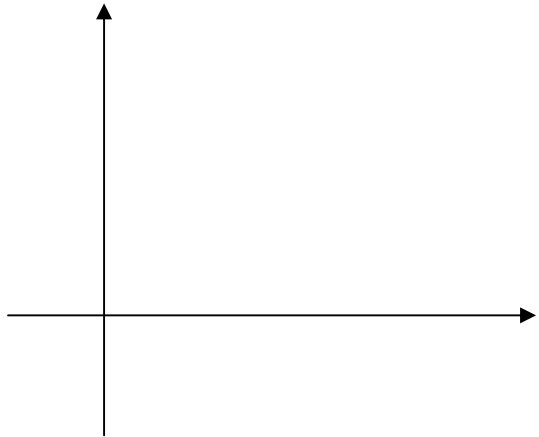
Displacement: _____

Total Distance Traveled: _____

Average Velocity: _____

Acceleration: Pos Neg Zero

Walk away and then toward:



Displacement: _____

Total Distance Traveled: _____

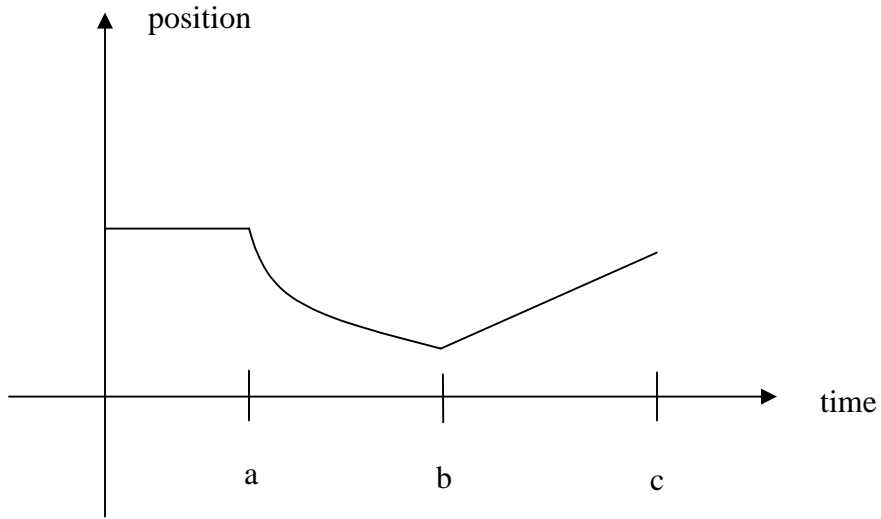
Average Velocity: _____

Walk A Graph

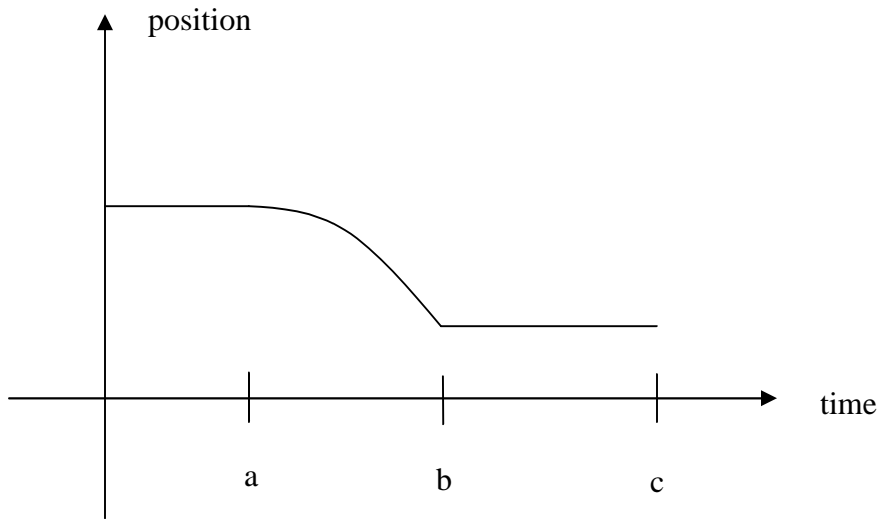
1. In this activity, you and a partner will attempt to reproduce a given position vs. time graph on the graphing calculator, using the CBR. To do this, connect a TI 84 Plus to a CBR2. This should activate the Easy Data program. Choose Setup, TimeGraph, Edit, 1, Next, 15, Next, and OK. Now, when you choose Start, the experiment will begin.
2. First, discuss with your partner, the motion required to produce the first position vs. time graph. Beside the graph given, write a description of the motion required, including:
 - whether the direction is away or toward
 - whether the average velocity is zero, constant but not zero, increasing or decreasing
 - whether the average acceleration is positive or negative
3. Second, have the walker walk the motion agreed upon while the other partner gathers their displacement vs. time data with the CBR2.
4. Next, compare the given graph with the graph produced by the walker.
5. Switch roles, and repeat #2 – 4 with a new given graph until all given graphs are completed.

(Graphs are on the next pages)

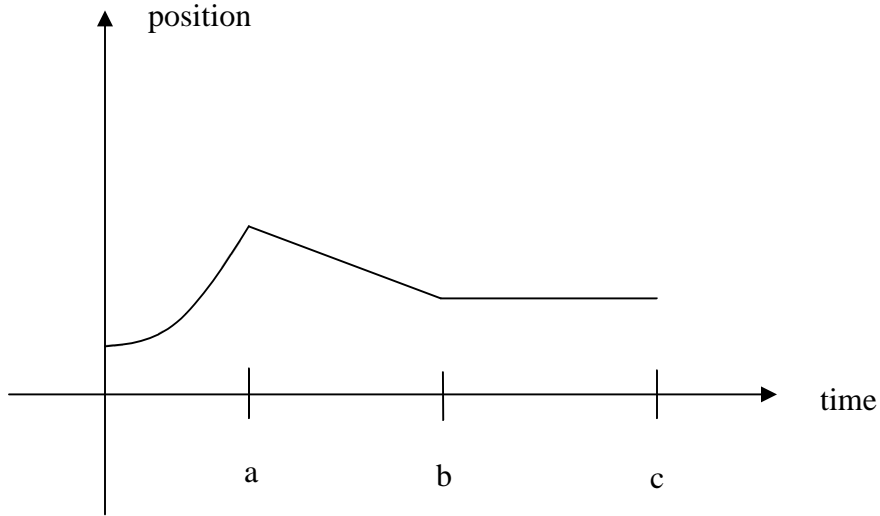
Graph 5:



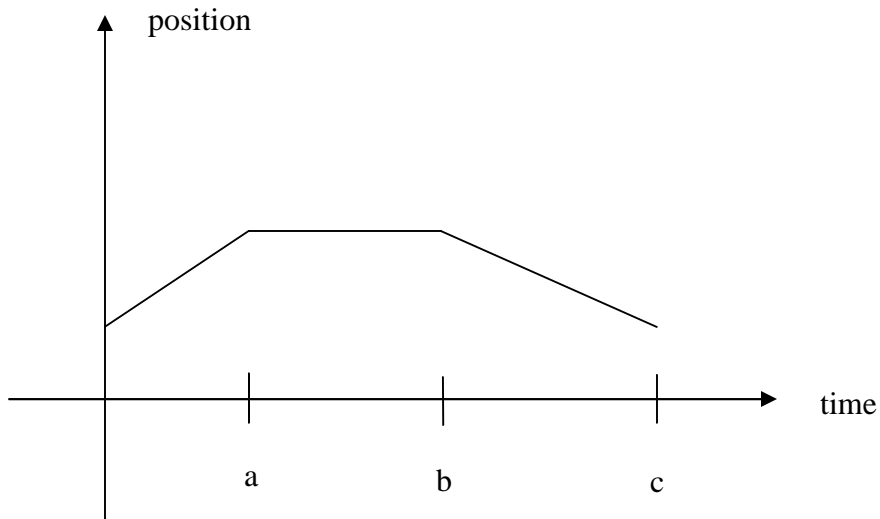
Graph 6:



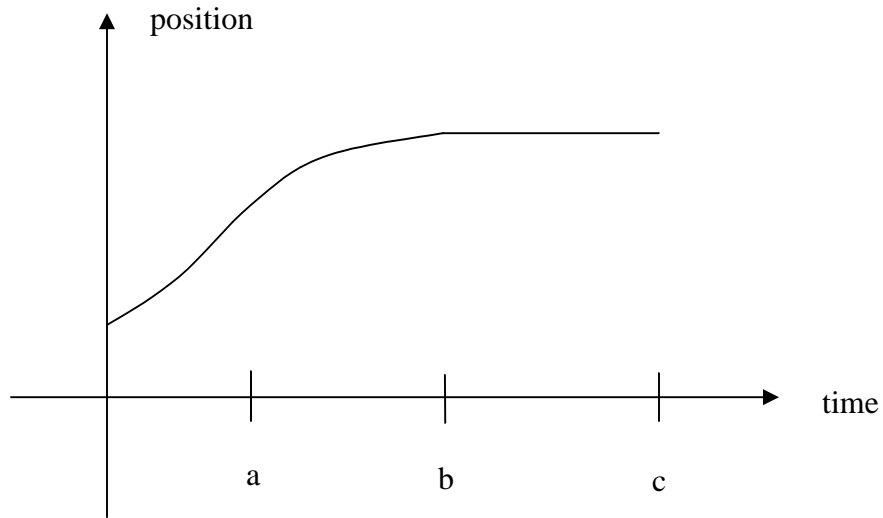
Graph 1:



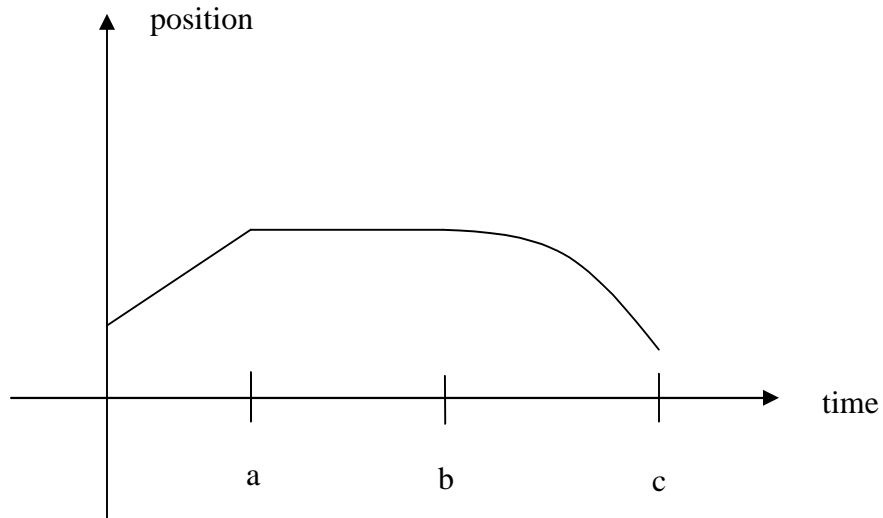
Graph 2:



Graph 3:



Graph 4:



Walk this Way 1

1. In this activity, you will use the Calculator Based Ranger 2 (CBR2) to examine the concepts of velocity and displacement in the context of one dimensional motion. To do this, connect your calculator to the CBR2 with a USB link. This should activate the calculator's Easy Data application and the Easy Data menu should show on the main screen of your calculator. Choose Setup, Time Graph, Edit, 0.5, Next, 12, Next, and finally, OK. Now, when you press Start the CBR2 will measure the distance between itself and an object in front of its circular screen in meters every $\frac{1}{2}$ a second for 6 seconds.
2. With your group members, mark off 5 meters. Have one group member hold the CBR2 and stand at the 0 meter mark. Have a second group member stand at the 1 meter mark. When Start is pressed on the calculator, have the second group member walk away from the CBR2 at a constant velocity.
3. Now you will need to transfer the data collected from this first walk to each group member's calculators. To do this, press **2nd LINK** and choose List. Scroll down till the cursor is beside L1 and choose it by pressing **ENTER**. Then scroll to L2 and choose it as well. Then move your cursor to the right to get ready to choose the TRANSMIT option at the top of the menu. Do not choose TRANSMIT until the receiving group member's calculator is ready. To ready the receiving calculator, press **2nd LINK** on the receiving calculator. Then move the cursor to the right and choose the RECEIVE option at the top of the menu. Once the RECEIVE option has been chosen on the receiving calculator, the TRANSMIT may be chosen on the sending calculator, and the data will be transmitted. Repeat this process so that all the group members' calculators have the data.
4. Each member of the group should now set up a plot of the data on their TI-84 Plus using the **STATPLOT** menu with option Plot 1. Within the Plot 1 menu, remember to turn on the plot, choose a scatter plot, choose L1 and L2 to be plotted, and choose the type of mark you'll use to indicate each point.
5. Using Transformation Graphing, to estimate a line of "good fit" for the data. Then record your line of "good fit" below.

$$y = \underline{\hspace{2cm}}$$

6. Repeat #2-5 for another walk, this time asking the walker to walk away from the CBR2 at a greater constant velocity. Record your line of "good fit" for this walk below.

$$y = \underline{\hspace{2cm}}$$

7. Repeat #2-5 for a final walk, this time asking the walker to start at the 5 meter mark, and walk toward the CBR2 at a constant velocity. Record your line of “good fit” for this walk below.

$y =$ _____

8. After you’ve found an estimated line of “good fit” for each data set, turn off Transformation Graphing by pressing **APPS**, choosing Transform and then choosing Uninstall. Then enter the equations of the estimated lines of “good fit” into your **Y=** menu in Y1, Y2, and Y3 respectively. Press **GRAPH** and sketch what you see below. Be sure to label and scale your axes.



Consider the parameters a and b value in the estimated lines of best fit. In your group, discuss the answers to the following.

9. For each of the data sets, what is the physical meaning of b ?
10. How is b related to the graph of the line?

11. For each of the data sets, what is the physical meaning of a (more commonly known in mathematics classes as m) ?

12. How is a related to the graph of the line?